

國立彰化師範大學112學年度碩士班招生考試試題

系所：統計資訊研究所(選考乙)

科目：統計學

☆☆請在答案紙上作答☆☆

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1. (5%) If X and Y are independently identically distributed gamma random variables with parameters $\alpha=2$ and $\theta=1/2$, find the expected value $E[2(X-1)^2(Y+1)^2]$.

Hint: Gamma distribution pdf

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-\frac{x}{\theta}}, \quad x > 0, \alpha > 0, \theta > 0.$$

$$\mu = \alpha\theta, \sigma^2 = \alpha\theta^2.$$

2. Let S be the three-element set $\{a, b, c\}$ and let A be the σ -field of all eight subsets of S . Define probabilities on these eight events by means of the following function:

$$P(\{a\}) = \frac{1}{7}, \quad P(\{b\}) = \frac{4}{7}, \quad P(\{c\}) = \frac{2}{7}.$$

Let X be the random variable from S to R defined by

$$X(\{a\}) = 1, \quad X(\{b\}) = 5, \quad X(\{c\}) = 0.$$

Find the following:

- (1)(5%) The probability $P(\{a, b\})$.
 - (2)(5%) The event $\{s: X(s) \in E\}$ where E is the interval $[3,6]$.
 - (3)(5%) The probability of the event in part (2), that is, $P(3 \leq X \leq 6)$.
 - (4)(5%) The probability of the event $\{s: X(s) = n\}$, that is, $P(X = n)$, for each $n = 0, 1, 2, 3, \dots$
3. Suppose that X and Y are independent random variables with X being uniform on $(0,1)$ and Y being exponential with mean 1. Let $U = XY$ and $V = X + Y$. Compute the following:
- (1)(5%) The covariance of U and V .
 - (2)(10%) The conditional density $f(u|y)$ of U given $Y = y$.
 - (3)(10%) The probability density function for V .

Hint: Exponential distribution pdf

$$f(y) = \frac{1}{\theta} e^{-\frac{y}{\theta}}, \quad y > 0, \theta > 0,$$

$$\mu = \theta.$$

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4. A random sample of size $n = 20$ is drawn from exponential p.d.f.

$$f(x) = \begin{cases} \frac{1}{\theta} \exp\left\{-\frac{x}{\theta}\right\}, & x > 0, \theta > 0 \\ 0, & \text{otherwise} \end{cases},$$

- (1) (10%) Find the p.d.f. of $X_{(1)} = \min\{X_1, \dots, X_{20}\}$.
 - (2) (10%) We want to test the null hypothesis $H_0: \theta = 10$ against the alternative hypothesis $H_1: \theta < 10$. The test is based on the critical region $C = \{X_{(1)} \mid X_{(1)} < k\}$ at a significant level of 0.05. Please find the value of k .
 - (3) (10%) What is the probability of a Type II error $\theta = 2$.
5. Consider the regression model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, $i = 1, 2, \dots, n$, where y_i 's are independent and distributed as $N(\beta_0 + \beta_1 x_i, \sigma^2)$.
- (1) (15%) Find the maximum likelihood estimators (MLEs) of β_0 and β_1 .
 - (2) (5%) Find the variance of the MLE of β_1 .