

國立彰化師範大學111學年度碩士班招生考試試題

系所：數學系(選考乙)、

科目：統計學

統計資訊研究所(選考乙)

☆☆請在答案紙上作答☆☆

共3頁，第1頁

1. (12%) 以下是三筆二維資料的散佈圖(scatter plot)，試從 -1.0 ， -0.81 ， -0.43 ， 0.22 ， 0.68 ， 0.97 六個數字當中，選擇三個數字(可複選)分別作為此三筆資料的相關係數。

圖 A

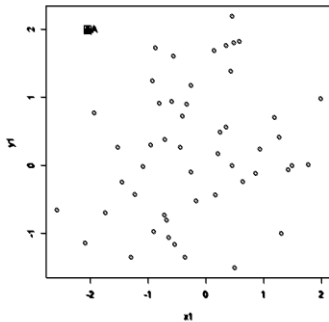


圖 B

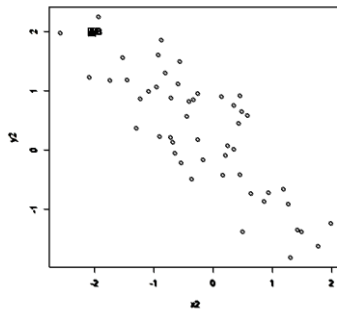
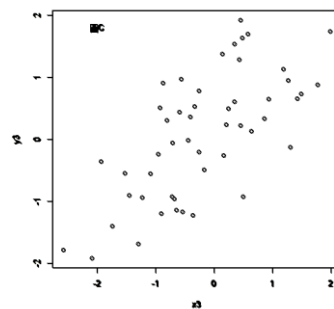


圖 C



2. (12%) The random variable X has probability density function

$$f(x) = \begin{cases} ax + bx^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

If $E(X) = 0.6$, find $P(X < 1/2)$, $\text{Var}(X)$, and the standard deviation of X .

3. (26%) Let X and Y be continuous random variables with the joint probability density function

$$f(x, y) = 2, \quad 0 \leq x \leq y \leq 1.$$

- (1) (3%) Let $A = \{(x, y): 0 \leq x \leq \frac{1}{2}, 0 \leq y \leq \frac{1}{2}\}$. Find the probability $P[(X, Y) \in A]$.
 - (2) (3%) Find $f_X(x)$, the marginal pdf of X .
 - (3) (3%) Find $f_Y(y)$, the marginal pdf of Y .
 - (4) (3%) Are X and Y independent? Explain.
 - (5) (8%) Compute the correlation coefficient of X and Y .
 - (6) (3%) Determine $f(y|x)$, the conditional pdf of Y , given $X = x$.
 - (7) (3%) Compute the probability $P\left(\frac{3}{4} \leq Y \leq \frac{7}{8} \mid X = \frac{1}{4}\right)$.
4. (30%) Let X_1, X_2, \dots, X_n be i.i.d. random variables from each of the following distributions, and please find the maximum likelihood estimator (MLE) of θ in each case.
- (1) (10%) Exponential distribution, $\text{Exp}(\theta)$.
 - (2) (10%) Uniform distribution, $U(\theta_1, \theta_2)$, $\theta_1 < \theta_2$.
 - (3) (10%) Bernoulli distribution, $\text{Ber}(\theta)$, $0 < \theta < 1/2$.

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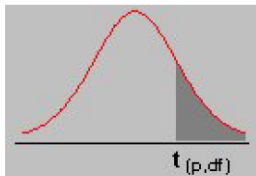
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共3頁，第2頁

5. (20%) Suppose that two independent samples of size 9 are selected from normal populations, one with mean μ_1 and variance σ_1^2 and the other with mean μ_2 and variance σ_2^2 .

	First population	Second population
Sample mean	35	31
Sample variance	24	20

- (1) (5%) Please construct a 95% confidence interval for the population mean μ_1 of the first population.
- (2) (5%) Please construct a 95% confidence interval for the population variance σ_1^2 of the first population.
- (3) (10%) Assume that $\sigma_1^2 = \sigma_2^2 = \sigma^2$, please test the null hypothesis $H_0: \mu_1 = \mu_2$ versus the alternative hypothesis $H_1: \mu_1 \neq \mu_2$ with significance level $\alpha=0.05$.



df\Pr	0.25	0.1	0.05	0.025	0.01	0.005
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947

df\Pr	0.25	0.1	0.05	0.025	0.01	0.005
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
19	0.688	1.328	1.729	2.093	2.539	2.861
20	0.687	1.325	1.725	2.086	2.528	2.845
21	0.686	1.323	1.721	2.080	2.518	2.831
22	0.686	1.321	1.717	2.074	2.508	2.819
23	0.685	1.319	1.714	2.069	2.500	2.807
24	0.685	1.318	1.711	2.064	2.492	2.797
25	0.684	1.316	1.708	2.060	2.485	2.787
26	0.684	1.315	1.706	2.056	2.479	2.779
27	0.684	1.314	1.703	2.052	2.473	2.771
28	0.683	1.313	1.701	2.048	2.467	2.763
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
inf	0.674	1.282	1.645	1.960	2.326	2.576

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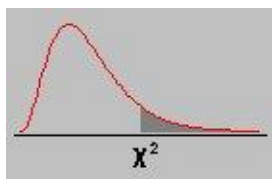
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df\Pr	$\chi^2_{0.995}$	$\chi^2_{0.99}$	$\chi^2_{0.975}$	$\chi^2_{0.95}$	$\chi^2_{0.9}$	$\chi^2_{0.1}$	$\chi^2_{0.05}$	$\chi^2_{0.025}$	$\chi^2_{0.01}$	$\chi^2_{0.005}$
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00