

# 國立彰化師範大學110學年度碩士班招生考試試題

系所：數學系(選考乙)、

科目：統計學

統計資訊研究所(選考乙)

☆☆請在答案紙上作答☆☆

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1. (25%) A random variable  $X$  with moment generating function (mgf)  $M_X(t) = e^{\mu + \frac{1}{2}\sigma^2 t^2}$ , where  $-\infty < x < \infty$ ,  $-\infty < \mu < \infty$ ,  $\sigma > 0$ ,  $-\infty < t < \infty$ . Define the random variable  $Y = \left[ \frac{(X - \mu)}{\sigma} \right]^2$ . Please use any technique to find the probability density function (pdf) of random variable  $Y$ .

2. (15%) Two independent samples are randomly selected from the normally distributed population A and B.

	Sample size	Sample mean	Sample variance
<b>Population A</b> (population mean: $\mu_A$ and variance: $\sigma_A^2$ )	10	531	192
<b>Population B</b> (population mean: $\mu_B$ and variance: $\sigma_B^2$ )	15	543.5	123

- (a) (5%) Test the null hypothesis  $H_0 : \sigma_A^2 = \sigma_B^2$  versus the alternative hypothesis  $H_1 : \sigma_A^2 \neq \sigma_B^2$  with significance level  $\alpha=0.05$ . State the decision rule and your conclusion.
- (b) (10%) Based on the result of (a), test the null hypothesis  $H_0 : \mu_A = \mu_B$  versus the alternative hypothesis  $H_1 : \mu_A \neq \mu_B$  with significance level  $\alpha=0.05$ . Calculate the p-value and state your conclusion.
3. (10%) Two linear regression model are constructed from the same samples:  
 Model A:  $E(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5$   
 Model B:  $E(Y) = \beta_0 + \beta_1 X_1 + \beta_3 X_3 + \beta_5 X_5$

ANOVA tables:

Model A	SS	DF	Model B	SS	DF
Regression	8500		Regression		
Error			Error	1550	
Total	10000		Total		30

SS: sum of square; DF: degrees of freedom

- (a) (6%) Calculate the multiple coefficient of determination ( $R^2$ ) and adjusted multiple coefficient of determination ( $R_{adj}^2$ ) for each model. Please compare model A and B based on the  $R^2$  and  $R_{adj}^2$ .
- (b) (4%) Please write the assumptions of the regression model.

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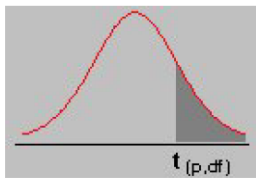
4. (30%) Let  $X_1, X_2, \dots, X_n$  be i.i.d. random variables from each of the following distributions, and please find the maximum likelihood estimator (MLE) of  $\theta$  in each case.

- (a) (10%) Bernoulli distribution,  $Ber(\theta)$ .
- (b) (10%) Poisson distribution,  $Poi(\theta)$ .
- (c) (10%) Uniform distribution,  $U(0, \theta)$ .

5. (20%) Let  $X_1, X_2, \dots, X_n$  be i.i.d. random variables from a distribution with mean  $E(X_i) = \mu$  and variance  $Var(X_i) = \sigma^2$ .

(a) (15%) Please prove  $E(\bar{X}) = E\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = \mu$  and  $E(S^2) = E\left(\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2\right) = \sigma^2$ .

(b) (5%) Please state the central limit theorem.



df\Pr	0.25	0.1	0.05	0.025	0.01	0.005
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947

df\Pr	0.25	0.1	0.05	0.025	0.01	0.005
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
19	0.688	1.328	1.729	2.093	2.539	2.861
20	0.687	1.325	1.725	2.086	2.528	2.845
21	0.686	1.323	1.721	2.080	2.518	2.831
22	0.686	1.321	1.717	2.074	2.508	2.819
23	0.685	1.319	1.714	2.069	2.500	2.807
24	0.685	1.318	1.711	2.064	2.492	2.797
25	0.684	1.316	1.708	2.060	2.485	2.787
26	0.684	1.315	1.706	2.056	2.479	2.779
27	0.684	1.314	1.703	2.052	2.473	2.771
28	0.683	1.313	1.701	2.048	2.467	2.763
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
inf	0.674	1.282	1.645	1.960	2.326	2.576

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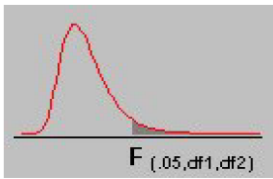
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df2/df1	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35