國立彰化師範大學109學年度碩士班招生考試試題

系所：統計資訊研究所(選考乙)　　科目：統計學

☆☆請在答案紙上作答☆☆　　

1. Let the random variable X have the probability mass function (pmf)

$$f(x) = \frac{(|x|+1)^2}{9}, \quad x = -1, 0, 1.$$

(1) Compute the values of the mean, variance and standard deviation of X. (8%)

(2) Find the moment-generating function of X. (5%)

(3) Compute $\mathrm{E}[2X^2 - 3X + 9]$. (5%)

(4) Compute $\mathrm{Var}\left(-\frac{X}{5} - 7\right)$. (5%)

2. A small convenience store has two checkout stations. Suppose that the joint probability mass function of the random variables X = number of customers at station 1 and Y = number of customers at station 2 is shown in the table.

		y		
	f(x, y)	0	1	2
x	0	.3	.08	.02
	1	.08	.2	.05
	2	.02	.05	.2

(1) Find the marginal probability mass function of X. (3%)

(2) Compute the conditional probability $\mathrm{P}(Y > 0.5 | X = 2)$. (3%)

(3) Compute the conditional expected value $\mathrm{E}(Y | X = 1)$. (6%)

(4) Compute the covariance and correlation between X and Y. (12%)

(5) Are X and Y independent? Why or why not? (3%)

3. Please prove the following questions.

(1) Let $U_1, \ldots, U_n$ be i.i.d. random samples from $U(-\theta, \theta)$ and $Y = \max(|U_1|, \ldots, |U_n|)$, please prove that $Y \xrightarrow{p} \theta$ when $n \to \infty$.(10%)

(2) Let $U_1, \ldots, U_n$ be i.i.d. random samples from $U(0,1)$ and $Z = \prod_{i=1}^{n} U_i$, please prove that $Z \xrightarrow{p} 0$ when $n \to \infty$.(10%)

4. Let $X_1,\ldots,X_n$ be i.i.d. random samples from a normal distribution, $N(\mu,\sigma^2)$, where μ and σ^2 are unknown. Please find the

   (1) method of moment estimator (MME) of $(\mu,\ \sigma^2)$. (10%)

   (2) maximum likelihood estimator (MLE) of $(\mu,\ \sigma^2)$.(10%)

   (3) minimal sufficient statistic for $(\mu,\ \sigma^2)$.(10%)