國立彰化師範大學108學年度碩士班招生考試試題

系所：數學系(選考乙)、統計資訊研究所(選考乙)　　科目：統計學

☆☆請在答案紙上作答☆☆　　

1. Let $X_1,\ldots,X_n$ be i.i.d. random samples from a two-parameter exponential distribution with

$$f(x)=\begin{cases}\frac{1}{\theta}\exp\left\{-\frac{(x-\mu)}{\theta}\right\}, & x\ge\mu\\ 0, & \text{otherwise}\end{cases}.$$

(1) Write down the likelihood function of (μ,θ) given the samples $x_1,\ldots,x_n$. (5%)

(2) For a fixed θ, please draw a graph for the likelihood function of μ. Then find the maximum likelihood estimator (MLE) of μ. (10%)

(3) If both two parameters are unknown, please find the MLE of (μ,θ). (10%)

2. Let $X_1,\ldots,X_n$ be i.i.d. random samples from a normal distribution $N(\mu,\sigma^2)$, where μ and σ^2 are unknown. Please find an unbiased estimator of

(1) μ (5%)

(2) σ^2 (10%)

(3) σ (10%)

3. Let $X_1,\ldots,X_n$ be i.i.d. random samples from a uniform$(0,\theta)$ with $\theta>0$.

(1) Please show that the maximum order statistic $X_{(n)}=\max\{X_1,...,X_n\}$ is sufficient for θ. (6%)

(2) Please find the probability density function of $T=\frac{X_{(n)}}{\theta}$. (7%)

(3) Based on the result of (2), please construct a $100(1-\alpha)$% confidence interval for θ. (7%)

4. Given the following definitions:

(1) Type I error and Type II error (4%)

(2) Significant level of a test (4%)

(3) Power function of a test (4%)

5. Let X_1,X_2,X_3 be i.i.d. random samples from a uniform$(\theta,12)$ and let $X_{(1)}=\min\{X_1,X_2,X_3\}$ be the smallest order statistic. Consider testing $H_0:\theta=0$ versus $H_1:\theta>0$, and H_0 is rejected if $X_{(1)}>4$.

(1) Please find the size of the test. (6%)

(2) Please find the power function of the test. (6%)

(3) What is the probability of type II error when $\theta=3$. (6%)