國立彰化師範大學106學年度碩士班招生考試試題

系所：統計資訊研究所　　科目：統計

☆☆請在答案紙上作答☆☆　　

1. Let X_1, X_2, X_3, X_4 be a random sample from $U(0,\theta)$ and let Y_4 be the largest order statistic of $\{X_1, X_2, X_3, X_4\}$, where the probability density function of Y_4 is given by

$$f_4(y;\theta)=\begin{cases}\frac{4}{\theta^4}y^3; & 0<y<\theta\\ 0; & \text{otherwise.}\end{cases}$$

Consider testing the null hypothesis $H_0:\theta=1$ against the alternative hypothesis $H_1:\theta>1$ and we reject H_0 if $Y_4\geq c$. Please answer the following questions. **(20%)**

(a) Find the constant *c* so that the significant level α is 0.05. (5%)
(b) Determine the power function of the test. (5%)
(c) Find the power of the test at $\theta=2$. (5%)
(d) Find the probability of Type II error when $\theta=2$. (5%)

2. Let $X_1,\ldots,X_n$ be a random sample with a common probability density function as follows:

$$f(x;\theta)=\begin{cases}\frac{2x}{\theta^2}; & 0<x\leq\theta\\ 0; & \text{otherwise.}\end{cases}$$

Suppose that $X_{(1)},\ldots,X_{(n)}$ are the order statistics of $\{X_1,\ldots,X_n\}$. **(30%)**

(a) Show that the largest order statistic $X_{(n)}$ is the maximum likelihood estimator of θ. (10%)

(b) Find the probability density function of $X_{(n)}$. (10%)

(c) Prove that $X_{(n)}$ is not an unbiased estimator of θ and find an unbiased estimator of θ based on $X_{(n)}$. (10%)

3. Consider the regression model:

$$Y=\beta_0+\beta_1X_1+\beta_2X_2+\beta_3X_3+\varepsilon,$$

where Y is the vector of responses, X_1, X_2 and X_3 are the corresponding independent variables for the regression coefficients β_1, β_2 and β_3. A sample of 24 observations is obtained and the results are summarized in the following table. **(30%)**

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Sequential Sum of Squares	Partial Sum of Squares
Intercept	1	0.166	0.094	1.76		
X_1	1	1.636	0.198	8.26	85.411	7.103
X_2	1	0.124	0.036	3.44	1.292	1.217
X_3	1	0.275	0.909	0.30	0.010	0.010

(a) Complete the following ANOVA table. (8%)

Analysis of Variance		
Source	DF	Sum of Squares
Model	(1) (2%)	(3) (2%)
Error	(2) (2%)	(4) (2%)
Corrected Total	23	88.794

(b) What are the coefficient of multiple determination and adjusted coefficient of multiple determination for the model, and please explain these two values? (6%)

(c) Please test the null hypothesis $\beta_1 = \beta_2 = \beta_3 = 0$, and write down the alternative hypothesis, the test used, and your conclusion using a 5% level of significance. (5%)

(d) To test $H_0 : \beta_1 = 0$. Please specify the test used, distribution of the testing statistic and your conclusion. Use $\alpha = 0.05$. (5%)

(e) Do the two explanatory variables X_2 and X_3 (consider together) have a statistically significant effect on the response Y ? Please specify the null and alternative hypotheses, test used, distribution of the testing statistic and your conclusion. Use $\alpha = 0.05$. (6%)

4. We wish to determine whether or not three different treatments produce different responses. Four observations are to be taken at each level of treatments, and the 12 total observations are to be run in random order. The data is shown here. **(20%)**

	Observation			
Treatment	1	2	3	4
A	5	5	4	5
B	6	6	1	2
C	2	2	1	2

國立彰化師範大學106學年度碩士班招生考試試題

系所：統計資訊研究所　　科目：統計

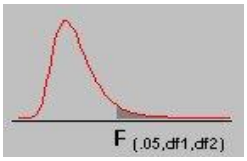
☆☆請在答案紙上作答☆☆　　

(a) Fill in the blanks (1)~(8) in the following ANOVA table. (14%)

Source of variation	Sum of squares	DF	Mean Square
Between treatments	(1) (2%)	(4) (2%)	(7) (1%)
Error (within treatments)	(2) (2%)	(5) (2%)	(8) (1%)
Total	(3) (2%)	(6) (2%)	

(b) Is there a difference in responses due to the treatments? Please specify the null and alternative hypotheses, the test used, the distribution of the testing statistic and your conclusion. Use $\alpha = 0.05$. (6%)

附表：



df2/df1	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18