國立彰化師範大學105學年度碩士班招生考試試題

系所：統計資訊研究所　　科目：統計

☆☆請在答案紙上作答☆☆　　

1. Let $X_1,\ldots,X_n$ be *i.i.d.* binary random variables with $P(X_i = 1) = \theta \in (0, 1)$ and $P(X_i = 0) = 1- \theta$. **(25%)**

(a) Find the maximum likelihood estimator (MLE) of $\theta(1-\theta)$. (10%)

(b) Find the uniformly minimum variance unbiased estimator (UMVUE) of $\theta(1-\theta)$. (10%)

(c) Is the UMVUE in (b) unique? Justify your answer. (5%)

2. Let $X_1,\ldots,X_n$ be a random sample from a uniform $(0, \theta)$ distribution with $\theta \in (0,\infty)$. **(25%)**

(a) Show that $X_{(n)}/\theta$ is a pivotal quantity, where $X_{(n)} = \max(X_1,\ldots,X_n)$. (10%)

(b) Construct a 100(1- α)% confidence interval for θ based on $X_{(n)}/\theta$. (10%)

(c) Give an explicit procedure to construct the shortest 100(1- α)% confidence interval for θ based on the result of (b). (5%)

3. Consider the following two regression models: **(35%)**

Model 1: $Y = \beta_0 + \beta_1 X_1 + \varepsilon$

Model 2: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$

where Y is the vector of responses, X_1 and X_2 are the corresponding independent variables for the regression coefficients β_1 and β_2, ε is the random error term and $\varepsilon \sim N(0,\sigma^2)$.

A sample of 20 observations was obtained, with the results summarized in the following tables.

Model 1		
Analysis of Variance		
Source	DF	Sum of Squares
Regression	(1)	2894.8
Error	(2)	(3)
Corrected Total	19	3144.9

Model 2		
Analysis of Variance		
Source	DF	Sum of Squares
Regression	(4)	3018.3
Error	(5)	126.6
Corrected Total	19	3144.9

(a) Please fill in the blanks (1) ~ (5) for the two ANOVA tables. (10%)

(b) To test $H_0 : \beta_1 = 0$ for the regression **model 1**. Please specify the test used, distribution of the testing statistic and your conclusion. Use $\alpha = 0.05$. (5%)

(c) Do the two explanatory variables X_1 and X_2 (consider together) for the regression **model 2** have a statistically significant effect on the response Y ? Please specify the null and alternative hypotheses, the test used, and your conclusion using a 5% level of significance. (7%)

(d) To test $H_0 : \beta_2 = 0$ for the regression **model 2** by using the two ANOVA tables. Please specify the test used, distribution of the testing statistic and your conclusion. Use $\alpha = 0.05$. (7%)

(e) What are the coefficient of multiple determination and the adjusted coefficient of multiple determination for the regression **model 2**. (6%)

4. We wish to determine whether or not four different treatments produce different responses. In addition, we have four blocking factors available and there is one observation per treatment in each block. Because there may be differences among blocking factors, we decide to conduct the experiment in a randomized complete block design. The ANOVA table for the randomized complete block design is summarized in the following table. **(15%)**

Source of variation	Sum of Squares	DF
Treatments	38.5	(2)
Blocks	82.5	(3)
Error	(1)	(4)
Total	129	(5)

(a) Fill in the blanks (1) ~ (5) in the above ANOVA table. (5%)

(b) Is there a difference in responses due to the treatments? Please specify the null and alternative hypotheses, the test used, the distribution of the testing statistic and your conclusion. Use $\alpha = 0.05$. (5%)

(c) Is there a difference in responses due to the blocks? Please specify the null and alternative hypotheses, the test used, the distribution of the testing statistic and your conclusion. Use $\alpha = 0.05$. (5%)

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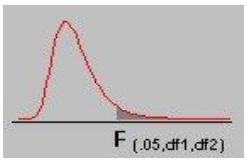
附表：



df2/df1	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18