系所：<u>數學系(選考甲)</u>、　　　　　　　　　　　　科目：<u>線性代數</u>
　　　<u>統計資訊研究所(選考甲)</u>

☆☆請在答案紙上作答☆☆　　　　　　　　　　　　　　　共 1 頁，第 1 頁

1. (20%) Find the solution set of the system of linear equations

$$\begin{cases} X_1 + 2X_2 - X_3 + 3X_4 = 2, \\ 2X_1 + 4X_2 - X_3 + 6X_4 = 5, \\ \qquad\quad X_2 + 2X_4 = 3. \end{cases}$$

2. (20%) Find an orthogonal matrix P and a diagonal matrix D such that $P^T AP = D$, where

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

3. (20%) Apply the Gram-Schmidt process to the subset $S = \left\{1, x, x^3\right\}$ of $P_3(\square)$ with the inner

   product

$$< f, g > = \int_0^1 f(t)g(t)dt$$

   to obtain an orthogonal basis β for $span(S)$.

4. (20%) Let B be obtained from A by a finite sequence of elementary row operations, where

$$B = \begin{pmatrix} 2 & 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & 3 & 2 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

   Determine A, if the first, second, and third columns of A are $\begin{pmatrix} 1 \\ -2 \\ -1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 1 \\ 2 \\ -4 \end{pmatrix}$, and $\begin{pmatrix} 2 \\ -3 \\ 1 \\ 2 \end{pmatrix}$, respectively.

5. (1) (10%) Find new coordinates x', y' so that the quadratic form $x^2 - 12xy - 4y^2$ can be written as

   $\lambda_1 (x')^2 + \lambda_2 (y')^2$.

   (2) (10%) Sketch the curve $x^2 - 12xy - 4y^2 = 40$.