

國立彰化師範大學109學年度碩士班招生考試試題

系所：數學系、

科目：線性代數

統計資訊研究所(選考甲)

☆☆請在答案紙上作答☆☆

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1. Let V denote the solution set of

$$\begin{cases} x_1 - x_2 + 2x_4 - 3x_5 + x_6 = 0, \\ 2x_1 - x_2 - x_3 + 3x_4 - 4x_5 + 4x_6 = 0. \end{cases}$$

(1) Find the solution set V . (10%)

(2) Show that $S = \{(0, -1, 0, 1, 1, 0), (1, 0, 1, 1, 1, 0)\}$ is a linearly independent subset of V . (10%)

(3) Extend S to a basis for V . (10%)

2. Find an orthonormal basis for the subspace $W = \text{span}\{(1, 2, 0, 2), (2, 1, 1, 1), (1, 0, 1, 1)\}$ of \mathbb{R}^4 . (15%)

3. Perform a rotation to eliminate the xy -term in the quadratic equation

$$3x^2 - 10xy + 3y^2 + 16\sqrt{2}x - 32 = 0 \quad \text{and} \quad \text{sketch the curve. (20\%)}$$

4. Let $T: P_2(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ defined by $T(p(x)) = p'(x) + p(2x-1)$, where $p'(x)$ is the derivative of $p(x)$ and $P_2(\mathbb{R})$ denotes the space of polynomials of degree at most 2 with real coefficients.

Let $\beta = \{x^2, x, 1\}$ and $\beta' = \{x^2 - 1, x + 1, x - 1\}$ be ordered bases for $P_2(\mathbb{R})$. Let $[T]_\beta$ and $[T]_{\beta'}$

be the matrix representations of T relative to β and β' , respectively. Find matrices $[T]_\beta$,

$[T]_{\beta'}$, and Q such that $[T]_{\beta'} = Q^{-1}[T]_\beta Q$. (20%)

5. Let $A = [A_{ij}]$ be an $n \times n$ matrix with characteristic polynomial

$$f(t) = (-1)^n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t + a_0. \quad \text{Prove that } f(t) = (A_{11} - t)(A_{22} - t) \cdots (A_{nn} - t) + q(t), \quad \text{where}$$

$q(t)$ is a polynomial of degree at most $n - 2$. (15%)