1. Use the $\varepsilon - \delta$ definition to show that $\lim\limits_{x \to 2} \dfrac{1}{x-1} = 1$.　(10%)

2. Let $f(x) = \dfrac{\ln x}{x}$, for $x > 0$. Show that $f$ attains its maximum at $x = e$.　(10%)

3. Show that $\int \dfrac{5x^2-3}{\sqrt[3]{(x^2-1)^2}} dx = 3x\sqrt[3]{x^2-1} + C$, where $C$ is a constant.　(10%)

4. Suppose $m \in \mathbb{R}$, $\Gamma_1 : y = mx$ and $\Gamma_2 : x = -y^2 + 2y$. Let $A$ be the area of the region bounded by $\Gamma_1$

   and $\Gamma_2$, and $B$ be the area of the region bounded by $\Gamma_2$ and the $y$-axis. Find $m$ such that

   $A = \dfrac{1}{2}B$.　(10%)

5. Show that $\sin\dfrac{1}{2} \leq \dfrac{1}{6} + \sin\dfrac{1}{3}$.　(10%)

6. Determine whether the given series converges or diverges by using any appropriate test.

   (1) $\sum\limits_{n=2}^{\infty} \dfrac{1}{(\ln n)^2}$　　(2) $\sum\limits_{n=1}^{\infty} \dfrac{n\,!}{n^4 e^n}$　　(10%)

7. Find the maximum and minimum values of $f(x,y) = 1 + y^2 - xy$ on the disk $x^2 + y^2 \leq 1$.　(10%)

8. Find the maximum and minimum values of $f(x,y,z) = 10 + 2xy + 4z$ on the circle that is the

   intersection of the plane $z = -x - y$ and the sphere $x^2 + y^2 + z^2 - 24 = 0$.　(10%)

9. Find the area of the finite plane region bounded by the four parabolas

   $y = \dfrac{x^2}{2},\ y = 2x^2,\ x = \dfrac{y^2}{3}$ and $x = 3y^2$.　(10%)

10. Evaluate $\iiint (x^2 + y^2)^2 dV$ over the first octant region bounded by the cylinders $x^2 + y^2 = 1$ and

    $x^2 + y^2 = 4$, the planes $z = 0, z = 2, x = 0$ and $x = y$.　(10%)