

# 國立彰化師範大學106學年度碩士班招生考試試題

系所： 統計資訊研究所

科目： 微積分

☆☆請在答案紙上作答☆☆

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1. Sketch the graph of the function  $f(x) = x^2 e^{-x^2/2}$ , mark (if they exist) local maxima and minima, inflection points and asymptotes. Be sure to express the concavity of the curve accurately. (10 %)

2. Find the 4<sup>th</sup> order Taylor polynomials at  $x = 0$  for the function  $f(x) = e^{-x^2/2} \cos x$ . (10 %)

3. Find the value of the following series  $\sum_{n=1}^{\infty} \frac{1}{n \cdot 5^n}$ . (10 %)

4. Determine the set of all  $x$  that makes the following power series converge, and explain why: (20 %)

(1)  $\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n \sqrt{n}}$ .

(2)  $1 + \frac{1}{2}x^2 + \frac{1 \cdot 3}{2 \cdot 4}x^4 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^6 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}x^8 + \dots$ .

5. Evaluate the indefinite integral  $\int \frac{x^2 + x + 1}{x(x^2 + 1)^2} dx$ . (20 %)

6. Let  $S_n = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1, x + y \geq 1/n\}$ , where  $n$  is a positive integer, and

$S = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$ . Show that the double integral  $\iint_{S_n} \frac{x-y}{(x+y)^3} dA = 0$  for all positive

integers  $n$ , but  $\iint_S \frac{x-y}{(x+y)^3} dA$  diverges. (30 %)